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2021-09

Kokkonen , T & Schalk , L 2021 , ' One Instructional Sequence Fits all? A Conceptual Analysis of the Applicability of Concreteness Fading in Mathematics, Physics, Chemistry, þÿand Biology Education ' , Educational Psychology Review , vol. 33 , nc

http://hdl.handle.net/10138/335059
https://doi.org/10.1007/s10648-020-09581-7

Educational Psychology Review (2021) 33:797–821
https://doi.org/10.1007/s10648-020-09581-7

REVIEW ARTICLE

One Instructional Sequence Fits all? A Conceptual Analysis of the Applicability of Concreteness Fading in Mathematics, Physics, Chemistry, and Biology Education

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Accepted: 28 October 2020/ Published online: 7 November 2020

Abstract
To help students acquire mathematics and science knowledge and competencies, educators typically use multiple external representations (MERs). There has been considerable interest in examining ways to present, sequence, and combine MERs. One prominent approach is the concreteness fading sequence, which posits that instruction should start with concrete representations and progress stepwise to representations that are more idealized. Various researchers have suggested that concreteness fading is a broadly applicable instructional approach. In this theoretical paper, we conceptually analyze examples of concreteness fading in the domains of mathematics, physics, chemistry, and biology and discuss its generalizability. We frame the analysis by defining and describing MERs and their use in educational settings. Then, we draw from theories of analogical and relational reasoning to scrutinize the possible cognitive processes related to learning with MERs. Our analysis suggests that concreteness fading may not be as generalizable as has been suggested. Two main reasons for this are discussed: (1) the types of representations and the relations between them differ across different domains, and (2) the instructional goals between domains and subsequent roles of the representations vary.

Keywords Concreteness fading · STEM learning · Multiple external representations · Learning processes

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To help students acquire, understand, and apply mathematics and natural science concepts, educators typically use and combine external knowledge representations. External representations come in various formats, such as texts, manipulatives, animations, sounds, pictures, graphs, or equations. Different external representations may serve different aims (Ainsworth 2006); they may also elicit different learning processes and thus affect what is learned (Belenky and Schalk 2014; Lampinen and McClelland 2018). Educational and psychological researchers have extensively examined how to present, combine, and sequence multiple external representations (MERs) to optimize learning.

A specific sequence that has gained considerable interest is referred to as concreteness fading. In this sequence, educators present concrete representations before increasingly abstract representations are introduced in a stepwise manner. Various researchers have suggested that the concreteness fading sequence is applicable widely across mathematics and natural science education (for overviews, see, Fyfe and Nathan 2019; Lin et al. 2016). While several studies have investigated the learning of basic primary school mathematics, only a few have investigated the learning of the more advanced concepts in mathematics, engineering, or the natural sciences. In the present conceptual analysis, we will scrutinize whether the concreteness fading sequence is indeed as broadly generalizable as suggested.

For this analysis, we integrate various strands of research from the psychological and cognitive sciences, research on learning and instruction, and domain-specific educational research focusing specifically on mathematics and natural science education. First, we will define external representations and emphasize that talking about sequences of representations in educational settings requires a precise distinction between external and internal knowledge representations. Second, we will state the obvious, that is, that realistic instructional settings always use and combine multiple external representations (e.g., graphs and formulas). However, this commonplace notion is highly important because different external representations and combinations thereof serve different functions. In order to frame the subsequent analysis, we will also propose that analogical and relational reasoning theories help to conceptualize and analyze the cognitive aspects of learning with MERs. Third, we will describe the concreteness fading instructional approach in more detail and describe how external representations can be classified. Fourth, we will conceptually analyze the applicability of the concreteness fading approach in mathematics and the natural science domains of physics, chemistry, and biology. To foreshadow the results of our conceptual analysis, the seemingly simple and straightforward idea of concreteness fading is appealing at first sight. However, we will identify profound differences between external representations used in mathematics and the natural sciences as well as profound differences between the domains with regard to the intended learning processes to be triggered by the MERs. These results thus put in doubt the broad applicability of the concreteness fading sequence.

External Representations and Concreteness Fading

Bruner (1965, 1966) introduced a category system for conceptualizing different external representations. He distinguished between enactive, iconic, and symbolic modes of representations (Bruner 1966, p. 11). Enactive representations are typically manipulatable objects triggering physical actions (i.e., motor responses). Iconic representations are graphical or pictorial visualizations. Symbolic representations are, as the name implies, symbol systems devoid of context; that is, they do not resemble their referents in the way that enactive or iconic representations do, using arbitrary symbols instead. This category system is still highly influential both in practice and in research on learning and instruction.

Bruner’s system also figures prominently in the concreteness fading approach (Fyfe et al. 2014; Fyfe and Nathan 2019; Goldstone and Son 2005). According to this approach, instruction should start with representations that are more concrete (e.g., enactive representations) and proceed to increasingly idealized representations in a stepwise fashion (e.g., symbolic representations). As an example, imagine a child who is being taught the concept of natural numbers. In the first step, the educator represents quantities using physical objects, such as apples. In the second step, the educator uses an iconic representation, such as a drawing of apples. In the third step, the educator introduces the symbolic representation, typically Arabic numerals. It has been suggested that this concreteness fading sequence improves learning and the ability to transfer knowledge better than the use of a single representation or a flipped sequence in which learners start with symbolic representations and then transition toward representations that are more concrete (Fyfe and Nathan 2019; Goldstone and Son 2005). What, however, are the precise ingredients of this sequence?

Defining External Representations

The concreteness fading sequence prescribes a specific order for presenting representations. We refer to these representations as external representations and define them as purposefully constructed objects in the environment that stand for some knowledge, concept, object, and so forth (cf. Zhang 1997). External representations can be understood as cultural tools with the aim of supporting thinking and reasoning (see, e.g., Tomasello 1999). Defining external representations as purposefully constructed objects implies three features. First, external representations are distinct from internal (i.e., cognitive) representations, such as concepts, mental models, or schemata. Internal representations reside in the individual, whereas external representations are entities or objects in the world. Specifically, internal representations of x are bodies of knowledge about x in an individual’s memory that are recruited and used by cognitive processes (Machery 2009). Second, defining external representations as purposefully constructed objects highlights the act of representing. Accordingly, the external representations are conceptualizations of the target knowledge or phenomenon presented in a different medium (i.e., a target cannot represent itself) with the aim of supplying knowledge (Pitkin 1967). Third, external representations are selective. That is, they are purposefully constructed to highlight specific aspects of a situation, a principle, or a theory, such as isolating some causal factors from others, as is typically the case in scientific modeling (Knuuttila 2011).

External representations serve as surrogates for reasoning (Knuuttila 2011; Kuorikoski and Ylikoski 2015). Because of our limited cognitive capacities, we regularly off-load processing demands onto the environment (Wilson 2002)—using an abacus to aid calculation or a computer to simulate complex natural phenomena, for example. This assistance in information processing makes external representations central to all sciences but also to education; if learners know how to “read” a diagram, it may be “worth 10,000 of any set of words” (Larkin and Simon 1987, p. 98). Given that external representations are selective, they are rarely used in isolation.

Multiple External Representations

External representations can support and constrain but may also hinder learning and reasoning in characteristic ways. That is, different external representations or combinations thereof affect what is processed and encoded (Knuuttila 2011; Lampinen and McClelland 2018; Louca and Zacharia 2012). For example, the use of graphs instead of tables leads to more flexible performance when interpreting data, but graphs can also induce interpretive bias (Braithwaite and Goldstone 2013). In realistic educational settings, it is rare to find a unit in which content is taught by using only one kind of external representation. Educators typically combine various types of external representations to trigger and support learning. Consequently, the design, use, and combination of MERs have received ample attention in mathematics and science education research during recent decades.

A guiding framework for this research was developed by Ainsworth (1999, 2006, 2008). Her Design-Functions-Tasks (DeFT) framework compiles a wide range of factors regarding the use of MERs. It covers the design parameters unique to MERs, (normative) functions that different MERs can have, and the ways in which different MERs elicit different cognitive tasks (Ainsworth 2006). The design parameters concern, for example, the number, form, and sequence of representations, which are also central considerations within the concreteness fading approach. The framework has been used in many studies across all STEM domains (for overviews, see Opfermann et al. 2017; Treagust and Tsui 2013).

The DeFT framework lays out three major functions for MERs (see Fig. 1). These functions are normative: they prescribe what external representations are supposed to do. First, MERs can have complementary roles. Different representations convey different pieces of information and support or hinder learning in characteristic ways. As already stated, graphs and tables facilitate different inferences. Here is another example: a macro-level representation of diffusion or electric current facilitates reasoning about the macro-level functioning of the system—such as calculating the concentration or electric current in different branches. By contrast, a micro-level representation does not straightforwardly allow this but helps in explaining how the macro-level properties emerge.

Second, MERs can constrain interpretation by virtue of familiarity or their inherent properties. For example, Lampinen and McClelland (2018) argue that different representations

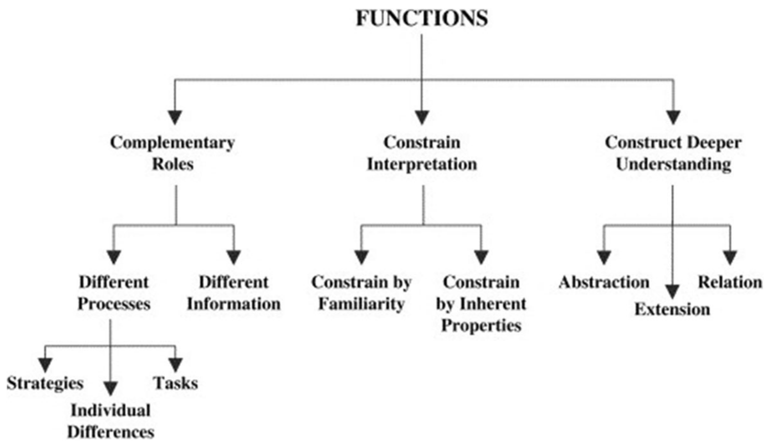


Fig. 1 The different functions of MER as conceptualized in the DeFT framework by Ainsworth (2006)

support the learning of complementary aspects of a target system (e.g., a scientific principle or a theory) by triggering different reasoning systems. Third, MERs can allow the construction of deeper understanding. Analogies, for example, can support the construction of abstract, higher-level mental representations that are more transferable (Richland and Simms 2015).

Fourth, the DeFT framework captures how cognitive tasks may influence the construction of and changes to internal representations depending on the function of the external representations. For instance, different modalities of external representations might elicit different cognitive processes, which in turn affect whether the external representations can be integrated productively (Ainsworth 2006). This description of learning is rather unspecific, however. It is necessary to describe precisely the learning processes that can be triggered when MERs are used in order to evaluate specific combinations of MERs, such as the concreteness fading sequence.

Learning from MERs

In the literature on the DeFT framework, and more specifically within the concreteness fading literature, cognitive aspects associated with learning from MERs have been discussed. However, only a few empirical studies have systematically examined these aspects or compared possible differences across domains. A systematic examination requires a unified framework. We suggest that theories of analogical and relational reasoning provide a frugal, yet powerful model for examining how combinations of different representations may trigger cognitive tasks, that is, learning processes.

Theories on analogical and relational reasoning, especially structure-mapping theory (Gentner 1983, 2010), have been developed to describe how combinations of basic cognitive processes are triggered by external representations. Structure-mapping theory postulates four basic, domain-general processes: abstraction, re-representation, inference projection, and difference detection. Abstraction denotes the integrated processing of two or more external representations that may lead to the extraction of a schema (i.e., an internal representation) capturing the commonalities of the external representations. Such a schema allows for knowledge generalization, that is, knowledge transfer. Re-representation denotes the change of an internal representation when a novel example (i.e., an external representation) does not fit to the "old" way of interpreting it. Inference projection denotes the generation of inferences from a well-understood to a less well-understood concept. Inference projection is, for example, supposed to happen when analogies or metaphors are used to explain something. Difference detection denotes the process of distinguishing two superficially similar but conceptually distinct external representations from one another. In structure-mapping theory, these four processes work in concert. However, different combinations or sequences of MERs may trigger some processes to a stronger extent than other processes.

The four processes of structure-mapping theory help to predict how combinations of external representations influence students' knowledge and competence development (i.e., changes in their internal representations). In emphasizing these processes, we do not want to suggest that there may not be other processes involved, but we do suggest that this collection of processes provides a parsimonious model to address the key question of which learning processes are triggered and to what degree by specific combinations of MERs. After having explained what we mean by external and internal knowledge representations, how they can be combined as MERs, and how to conceptualize learning from MERs with structure-mapping theory, we can start to scrutinize concreteness fading as a specific instructional approach to sequencing MERs.

Defining Concreteness Fading

External representations vary along many dimensions. For example, they may vary in their dimensionality (2D vs. 3D), abstractness, or dynamism (Ainsworth 2006). They may afford different sensory channels (visual/auditory/haptic). They may be more or less familiar for the learner or be embedded in more or less familiar (narrative) contexts. Put simply, there are many different external representations, varying along various dimensions and potentially eliciting very different processes.

The concreteness fading approach focuses on one dimension along which external representations differ: concreteness. Fyfe and Nathan (2019) attempt to provide a rigorous definition of concreteness fading as an instructional sequence. They argue that a lack of specificity may have limited research and application in educational practice. They define concreteness fading as "the three-step progression by which a concrete representation of a concept is explicitly faded into a generic, idealized representation of that same concept" (Fyfe and Nathan 2019, p. 411). It is assumed that variations along the dimension of concreteness influence students' learning. On the basis of this assumption, the approach further specifies how external representations differing in concreteness should be sequenced, and how many should be sequenced. To be sure, there exist alternative descriptions of concreteness fading sequences that differ from this definition; for example, with regard to whether two vs. three steps are necessary (Goldstone and Son 2005; Jaakkola and Veermans 2018; Johnson et al. 2014; Lin et al. 2016). Nevertheless, the different definitions converge on a common idea.

The concreteness fading sequence aims to exploit the different benefits of MERs varying in concreteness. The sequence starts by offering a grounded, concrete external representation. Ideally, this representation should be physical (i.e., enactive), but if it is not feasible to implement a physical representation, the initial representation should at least resemble a physical representation (Fyfe and Nathan 2019). It has been argued that these representations enhance learning through the activation of relevant real-world knowledge, the support of reasoning by inducing physical or imagined action, and the provision of meaningful and engaging learning experiences (e.g., Fyfe and Nathan 2019; Glenberg et al. 2004; Goldstone and Son 2005). Subsequently, more and more idealized external representations are introduced stepwise.

Within structure-mapping theory, the initial representation can be regarded as a base analog from which inferences can be projected to a subsequent, less concrete representation—the target analog. The target analog in concreteness fading sequences is typically a symbolic external representation. If symbolic representations are understood correctly, the ability to transfer knowledge will increase because the symbols are not tied to or grounded in a specific context but are idealized to be applicable broadly. This idealization directs learners' attention to the deep, relational aspects whereas (more) concrete representations can sometimes hinder generalization because they contain (more) irrelevant or even seductive details (Belenky and Schalk 2014; Fyfe et al. 2014; Goldstone and Son 2005; Kaminski et al. 2008). Intermediate representations—for example, iconic representations—serve as a bridge between the concrete base and the idealized target. For example, the concept of "one quarter" can be first introduced with a pizza cut into four equal slices (in Bruner's words, this would be called an enactive representation).

The next step could be a pie chart divided into four parts (iconic). Finally, the numeric representation ¼ is presented (symbolic). The term fading thus refers to the fact that each successive external representation is stripped of perceptual information that is extraneous to the target. This fading should help learners to understand the generalizable symbolic representation with the help of their prior knowledge via inference projection. Moreover, if learners structurally integrate the different external representations, this may support the abstraction of a generalizable schema. Thus, concreteness fading may be a sequence that optimally combines the benefits of MERs.

Frameworks for Classifying External Knowledge Representations

We have used the terms concrete and idealized to describe external representations, but what does it mean when an external representation is referred to as concrete or idealized? As mentioned above, Bruner (1966) distinguishes between enactive, iconic, and symbolic representations. These terms provide quite sharp category boundaries. By contrast, many studies discussed in the concreteness fading literature have focused on pictorial representations or computer simulations, subtly varying the degrees of concreteness.

Fyfe and Nathan (2019) argue that all external representations are concrete but vary in terms of how idealized they are. Accordingly, external representations vary in their amount of perceptual and conceptual information, specifically with regard to their familiarity and perceptual richness. The more idealized an external representation is, the easier it should be to infer the invariant relation of a target concept or principle. In addition to this nuanced description of how external representations differ, Fyfe and Nathan state (2019, p. 410):

> […] we use the term concrete representation to refer to any external representation. We distinguish types of concrete representations by referring to them as more or less idealised. […] We acknowledge that this is still somewhat unsatisfactory (e.g. one representation could be more idealised on a perceptual dimension but less idealised on a conceptual dimension), but we believe this highlights the true nature of external representations as concrete representations that vary in the amount of information they convey to a person interacting with them for some contextualised purpose.

Belenky and Schalk (2014) have suggested a somewhat similar organizing framework wherein external representations are ordered along the dimensions of groundedness and inclusion of irrelevant details. The dimension of groundedness captures the extent to which concepts or principles are contextualized. For example, bringing a pizza cut into equal slices into the classroom to explain fractions would be considered a more grounded external representation than showing a picture of a pizza cut into equal slices. A picture of a pizza is, to some extent, an abstraction (that does not necessarily correspond to any particular real pizza) constructed to represent the target concept and is therefore a less grounded representation than a real pizza. The dimension of irrelevant details captures the extent to which an external representation contains details that are unrelated to the concept or principle that the educator wants to convey. For example, imagine that an educator wants to illustrate the concept of "one quarter" by holding up a quarter slice of a pizza. However, the pizza is topped with pieces of pepperoni, and accidentally, every quarter of the pizza is topped with exactly four pieces of pepperoni. The pieces of pepperoni are irrelevant details. In this example, these details might even mislead learners to think that "one quarter" is a slice of a pizza topped with four pieces of pepperoni.

We repeat that, regardless of the framework used to describe external knowledge representations, there are obviously many ways to represent concepts or principles.

Not only are there many different ways of representing concepts or principles, but also, as already mentioned, different disciplines may also rely on or prefer different representations. Lin et al. (2016) notice these differences and develop a taxonomy for extending the concreteness fading sequence from mathematics to different natural science disciplines. Table 1 provides this taxonomy, capturing the four disciplines of mathematics, physics, chemistry, and biology. This taxonomy is conceptually based on the chemistry triplet (Johnstone 1991; Talanquer 2011). That is, the least idealized level, typically the concrete, grounded, and/or enactive examples in mathematics, corresponds to the macro scale of objects or phenomena. On the macro scale, "natural phenomena can be observed through the senses (sight, touch, etc.)" (Lin et al. 2016, p. 645)—such as watching water melt or dye diffuse into a liquid. The symbolic representations are the chemical symbols and equations. This most idealized level of representation in the natural sciences seems to correspond well with symbolic notations in mathematics. The intermediate level in the taxonomy consists of schematic representations of atoms and molecules at the nano scale (or micro/sub-micro scales) in the natural sciences. This intermediate level, in contrast to symbolic representations, is less obviously aligned with the intermediate examples used in mathematics (faded examples in Lin et al.'s taxonomy).

Comparing the levels across disciplines makes it evident that there are always three external representations (corresponding to the definition of concreteness fading in Fyfe and Nathan 2019), but these vary along more than one dimension. That is, progressing from concrete (or less idealized) to (more) idealized external representations within mathematics cannot be directly and easily aligned with the progressions from the macro scale to intermediate schematic representations used widely in physics. For example, a force diagram in physics may be seen as a schematic representation of a concrete example (e.g., a small car hitting a truck). The diagram typically consists of an idealized line drawing stripped of perceptual details. This representation would have to have properties similar to those of the pictorial representation of apples in our mathematics example above (i.e., stripping away contextual detail). However, while force diagrams reduce contextual or perceptual detail, they typically also introduce something new, since vector representation of forces is not present in the macro-scale representation. Lin and colleagues' taxonomy is by no means the only possible way to conceptualize concreteness fading across disciplines, but it clearly indicates how the basic idea of concreteness fading—varying representations along the dimension of concreteness—might align different representations used in different disciplines. Taking the definition of Fyfe and Nathan (2019), the taxonomy by Lin et al. (2016), and existing studies about concreteness fading as starting points, we pose a general question for the present conceptual analysis: Can the basic idea of concreteness fading in principle be used as a domain-general instructional strategy? Specifically, would following a sequence from top to bottom always be the more beneficial than following a sequence from bottom to top or using a single representation, as

Table 1 Extended taxonomy of different levels of concreteness for different disciplines (Lin et al. 2016)

Mathematics	Physics	Chemistry	Biology
Concrete example	Macro scale	Macro scale	Macro scale
Faded example	Invisible (forces)	Nano scale	Micro scale
Symbolic notation	Symbolic	Symbolic	Biochemical

should be the case if concreteness fading is an effective domain-general instructional sequence?

Concrete Examples of Concreteness Fading

We will evaluate this question by conceptually analyzing both existing and possible examples of concreteness fading sequences in the domains of mathematics, physics, chemistry, and biology. Our aim is to compare not only the specific implementations in these four domains but also to describe the types of external representations, their qualities, and how they relate to each other, as well as the functions and tasks they play in these implementations (i.e., which learning processes educators want to trigger, normatively, when using these representations). All the existing examples that we examine in the following were labeled as concreteness fading by their authors. Only a few of the studies perfectly fit the definition of concreteness fading given by Fyfe and Nathan (2019), but all examples adhere to the general idea of fading from more concrete to less concrete external representations.

Mathematics

As already mentioned in the introduction, most of the concreteness fading research literature has investigated implementations in mathematics education. We will start by describing several studies and their results before we analyze the representations used and the intended learning processes.

A close fit to the concreteness fading definition provided by Fyfe and Nathan (2019), which we described above, is a study by Fyfe et al. (2015), who have second and third graders learn the mathematical concept of equivalence under four conditions: concrete, idealized, concreteness fading, and concreteness introduction. In the concreteness fading condition (see Fig. 2a, b), the children first learn the concept with either block-and-balance scales or puppets and stickers (concrete), then with respective pictorial representations (intermediate), and finally with symbolic equations (idealized). In the concreteness introduction condition, the children learn using the sequence in reverse (i.e., idealized-intermediate-concrete), whereas, in the concrete and idealized conditions, the children learn with only one type of representation. Children in the concreteness fading condition show the best transfer performance. In another study, Ottmar and Landy (2017) teach algebra to seventh graders using software that presented mathematical symbols as movable tangible objects. With this software, the simplifying of algebraic expressions involved the moving of symbols presented as colored tiles on a computer screen from one place to another. When, for example, factoring $2x + 3x$ into $x(2 + 3)$, the common symbols can be dragged and dropped on top of each other. In one condition, which the authors refer to as concreteness fading, the students receive instruction with the dynamic, movable tiles first and with static images second. In another condition, the order is reversed. The immediate post-test indicates superior performance for learners in the concreteness fading condition but not in the subsequent follow-up test after 1 month.

In another set of studies, undergraduates were taught the mathematical concept of the commutative group of order three (De Bock et al. 2011; Kaminski et al. 2008; McNeil and Fyfe 2012). The concrete representations instantiated the concept using images of either measuring cups containing different amounts of liquid, pizzas having different numbers of

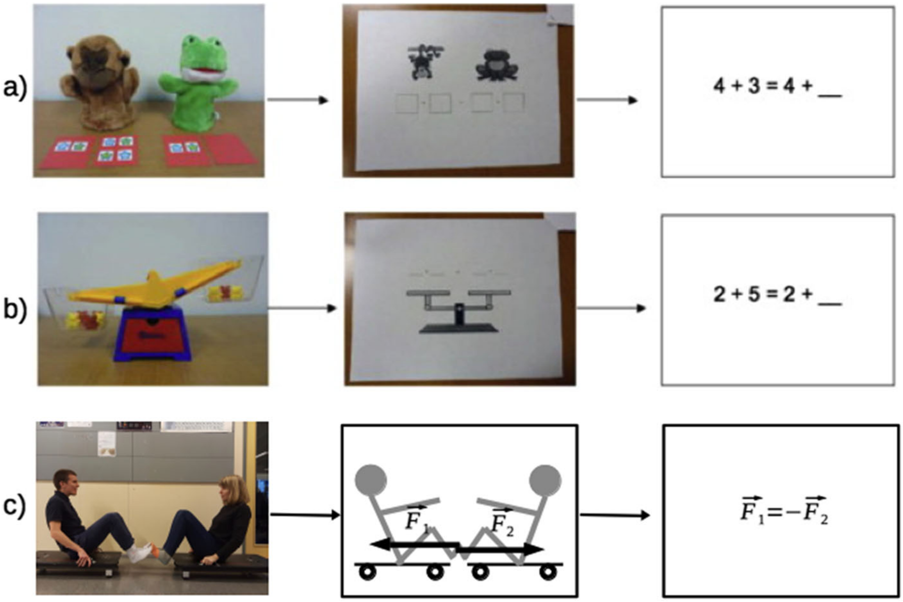


Fig. 2 **a**, **b** Concreteness fading sequences for teaching the concept of equivalence in mathematics (Fyfe et al. 2015), and **c** a potential sequence for teaching Newton's third law

burned slices, or batches of tennis balls. Generic but not mathematical symbols (such as circles or rhombi) were used as the idealized representations. Kaminski et al. (2008) conclude that learning with only the generic symbols of the target concept results in better transfer than does a concreteness fading sequence. However, these results are qualified by a replication by De Bock et al. (2011) with a more comprehensive transfer assessment. They report that transfer from learning with concrete representations to concrete transfer tasks or from learning with idealized representations to idealized transfer tasks is better than across representations. In this replication, concreteness fading (composed of a concrete representation followed by an idealized representation) leads to worse transfer performance than seeing sequentially either two concrete representations, two idealized representations, or a concreteness introduction condition (an idealized representation followed by a concrete representation). McNeil and Fyfe (2012) conduct another conceptual replication of the study by Kaminski et al. (2008) while also linking the concrete and the idealized representations by an intermediate representation (i.e., adhering to the three-step definition of concreteness fading). This intermediate representation uses roman numerals to represent, for instance, the proportions of liquid in measuring cups. McNeil and Fyfe (2012) compare a generic-only condition, a concrete-only condition, and a concreteness fading condition. Concreteness fading results in the best transfer performance.

This collection of concreteness fading studies in the domain of mathematics used quite different external representations. Nevertheless, there is a strong commonality: all external representations were strictly analogous. In fact, they were isomorphic, that is, there were stable one-to-one correspondences between the elements of the representations. This isomorphism may support analogical mapping, because it makes distinct representations structurally consistent (Holyoak and Thagard 1997). By adding more perceptual detail to a representation, it is made more concrete; by stripping away details, it is made less concrete. However, the structural similarity remains invariant across the representations. Fyfe and Nathan (2019) argue that concrete representations help to make sense of initially vague idealized representations. This sense-making can be conceptualized as inference projection, whereby the learner uses the previous examples to generate inferences to subsequent ones (Gentner 2010). The representations used in these mathematics examples may also support schema abstraction, that is, the process of acquiring a common schema that underlies all external representations (Gentner 2010; Goldwater and Gentner 2015). Moreover, the most idealized representation makes this schema salient, especially if represented with the notation that mathematicians might use. Indeed, understanding mathematical notation and its generality is the typical learning goal of the implementation of the concreteness fading sequence in mathematics. The less idealized, grounded, or enactive representations are rather arbitrary; one could use pizzas, cakes, measuring cups, and so on to represent the target concept. Thus, a concreteness fading sequence in mathematics builds on highly analogous external representations (the different steps in the sequence strip away details), and its goal is to help learners understand mathematical symbolism.

There are well-known caveats in using analogies in teaching. One is that problems or examples that are isomorphic (i.e., implementing similar mathematical principles) may not appear as such to students—for instance, nonessential surface features of the problems (e.g., the cover story of the problem) may distract students (see, e.g., Richland and McDonough 2010). While the mathematics examples of concreteness fading include quite close analogies that might even share surface features (e.g., an apple vs. a picture of an apple), students still sometimes fail to transfer across representations (cf. de Bock et al. 2011), that is, they do not recognize the underlying common principle. In other words, the initial representations sometimes do not to trigger the prior knowledge as intended by the educator. Therefore, researchers have suggested using multiple analogies (Richland and McDonough 2010). This suggestion is in line with the view of Fyfe and Nathan (2019) that a three-step sequence is better than a two-step sequence, although no study has directly examined this hypothesis. Using an intermediate representation may facilitate the alignment of the initial representation with the idealized one, but it remains to be seen whether the initial representation triggers useful prior knowledge and whether the three representations are easily alignable in such a way as to facilitate the learning of the generic principle underlying the examples as anticipated by the educator. In what follows, these questions will become more apparent when we analyze implementations of the concreteness fading approach in other disciplines.

Physics

The research literature on the concreteness fading sequence features a number of examples of its implementation in physics education. In one study, Jaakkola and Veermans (2018) use simulations to teach the basic concepts of direct current (DC) circuits to primary school students. Their concreteness fading condition is based on two simulations: first, a "concrete" simulation with schematic representations of bulbs, and second, an "abstract" simulation with resistors instead of bulbs. Students are assigned either to a concrete-only or a concreteness fading condition. In the concrete-only condition, students examine simulations with bulbs only, while in the concreteness fading condition, they start with the simulations with bulbs and subsequently work with the simulation with resistors. Learning is compared across these two conditions for three grade levels (fourth, fifth, and sixth). In a post-test that does not require transfer, fifth graders in the concrete-only condition perform better than the fifth graders in the concreteness fading condition. However, there are no significant differences for fourth and sixth graders and no differences at all for additional transfer tasks. The non-significant differences are not interpretable; they may simply indicate a lack of statistical power. In addition, Jaakkola and Veermans (2018) report the time spent for learning. Students are faster under the concrete-only condition. Thus, there is no clear-cut advantage in using concreteness fading sequences when learning about DC circuits.

In another study, Johnson et al. (2014) investigate students' procedural learning of electrical circuit problems. The idealized representations are conventional electrical circuit diagrams; the concrete representations embed the circuits in pictures of real-life settings using circuit components from everyday devices, such as batteries, bulbs, and other household appliances. Johnson and colleagues report that presenting conventional circuit diagrams before contextualized ones (i.e., concreteness introduction) produces better near transfer than does the reverse order (i.e., concreteness fading) or when only the concrete representations are presented. The concreteness introduction condition also results in far better transfer than does the concrete-only condition. No difference is reported in relation to the idealized-only condition. This study thus indicates disadvantages of the concreteness fading sequence.

Despite these empirical doubts about the effectiveness of the concreteness fading sequence in physics, we believe it is even more important to highlight the fact that the way concrete representations are conceptualized in the theoretical works of Fyfe and Nathan (2019) and Lin et al. (2016) is not fully reflected in their use in the above studies. To illustrate this difference, we constructed a fictitious implementation based on these theoretical works—a sequence for learning about Newton's third law. Figure 2 juxtaposes this three-step sequence with the concreteness fading sequences in mathematics as used by Fyfe et al. (2015). The first representation of the fictitious physics implementation is enactive and concrete. It aims to show the idea that interaction always has two parties, that is, both carts in which the persons sit move regardless of whether either one of the two persons or both are pushing. The second step introduces a diagram of the carts as well as vectorial representation of the forces. In the third step, Newton's third law is depicted as an equation stripped of any contextual details.

In line with the theoretical accounts of the concreteness fading approach, the fictitious example progresses from an enactive, concrete representation (or experience) to a highly idealized symbolic representation. This progression is bridged by the intermediate diagrammatic representation, which consists of a visualization of the concrete objects as well as representations of the "invisible" forces and fields depicted as vectors. So how does our fictitious implementation correspond to the two physics studies described above and to the mathematics examples presented earlier? While the simulations are not physically manipulatable or tangible (and not concrete in that sense), the "real-life" circuit diagrams (Johnson et al. 2014) and simulations with bulbs (Jaakkola and Veermans 2018) contain perceptual details and additional information that ground and contextualize the target concepts. In this sense, these concrete representations and our enactive representations provide a starting point for concreteness fading that seems to match with the mathematics examples. However, the representations used in our fictitious example and the two DC circuit examples have different properties than the representations used in the mathematics examples. According to Fyfe and Nathan (2019), idealized representations make the invariant relation more salient by dropping irrelevant details. However, simulations with bulbs, while introducing perceptual richness, also provide additional relevant information (i.e., the bulbs' brightness). In our example of Newton's third law, one could embed a vector diagram into a picture of the two carts. The trade-off would then be that the tangibility of the initial representation is lost (as one would switch from a physical experiment to a picture). Moreover, the concrete physics examples are highly specific, detail-rich instantiations of certain physics laws, models, or principles whereas, in mathematics, one has much more freedom to construct concrete representations and, at the same time, more control over the level of detail to be included. In other words, while the additional details of the concrete physics examples are not part of the invariant relation depicted most readily by mathematical symbolism, they necessarily have to be considered when evaluating the applicability of the principle or law. Moreover, in mathematics, the concrete materials instantiate the generalizable concept (i.e., concrete examples are constructed to illustrate the principles), whereas in physics, the relation is the other way around: A principle or law is thought of as a representation of certain phenomena or experiments. We will explain this crucial difference in more detail below.

The DC circuit representations offer obvious alignable similarities but are not isomorphic in the same sense as the external representations used in mathematics. Still, the perceptual similarity of the simulations may facilitate mapping between them. There is indeed some evidence that comparison of, for instance, real circuits and a simulation supports learning of the relevant concepts (Jaakkola et al. 2010). According to structure-mapping theory (Gentner 2010), comparisons can support students in abstracting a common schema as well as projecting inferences from one internal knowledge representation to another. Moreover, in the DeFT framework of Ainsworth (2006), additional details (such as glowing bulbs and flowing electrons) constitute complementary information relevant to the target concept. Visual representations of the electric current flow might invite inference projections to the real circuits as well as re-representation of internal knowledge representations (e.g., enriching the mental model) of an electric current.

In examples that are more complex, such as the fictitious physics example, the relationship between the external representations is different than the relationship in mathematics. Recognizing a phenomenon or a problem as an instantiation of Newton's third law, for instance, is a real insight problem. Typically, this recognition of the deep features of a problem requires several steps (Hestenes 1992; Savelsbergh et al. 2011). In the fictitious example (Fig. 2c), students may first have to notice that they need to learn something about an interaction. Then, they would have to identify the relevant interacting objects (the cart-person systems), their relevant properties (e.g., that color, shape, etc., do not matter whereas mass can be relevant), and identify the relevant interactions among the objects and their properties, as well as their magnitudes and directions.

In mathematics, the concrete and intermediate representations are, as we mentioned already, rather arbitrary. They serve mainly to make an idealized representation of a mathematical concept understandable. In our fictitious physics example, however, all three representations are of importance—and we believe, this holds true for physics in general. The goal of physics educators is for their students to understand physics principles. Physics experiments need to be designed carefully, so that they can demonstrate a concept unequivocally. Students have to learn and understand the specificities of such an experimental set-up. Students have to understand diagrammatic representations, such as a force diagram, as well as how to model these complex situations using mathematical symbolisms. These competences require comprehension of the particular external representations and awareness that these different representations complement each other. In other words, learners need to abstract the schemata of physics concepts, inference projections have to go back and forth between representations, differences between representations have to be detected, and learners need to re-represent physics concepts, such as the concept of force, as in our fictitious example, to overcome their naïve conceptions. In sum, representations play different roles in physics and mathematics, and the educational goals differ.

Chemistry

In chemistry, the ubiquitous triplet forms a progression that initially looks similar to the concreteness fading sequences of mathematics—at least when aligned as in the taxonomy of Lin and colleagues (see Table 1). The chemistry triplet (Johnstone 1991; Talanquer 2011) comprises three different kinds of external representation: macro, micro, and symbolic. There has been considerable discussion about the interpretation and viability of Johnstone's original ideas (e.g., Taber 2013; Talanquer 2011), but it is possible to roughly define the triplet's levels of explanation. The macro level corresponds to the level at which phenomena are "experienced, observed and described" (Talanquer 2011, p. 180; see also Johnstone 1991). For example, two substances are mixed and the student observes the reaction, or students are asked to describe different materials and substances in terms of their macroscopic properties, such as density and weight. Some researchers equate the macro level with actual observable phenomena, while others also include the concepts that are used to describe them (Talanquer 2011). At the micro level, the macro-level phenomena are represented in terms of molecular and/or atomic models of chemical reactions. The micro level is further distinguished by some authors as multi- or single-particle, molar, molecular, or electrical representations. These distinctions depend on whether the chemical properties to be explained refer to properties of the atoms or to groups of atoms, density and state of matter being the most basic examples (Talanquer 2011). Finally, the symbolic level comprises chemical symbols, formulas, and equations "used to represent the properties and behavior of chemical substances and processes" (Talanquer 2011, p. 184).

Lin et al. (2016) arrange the triplet along a continuum of concreteness. The macro level is the most concrete (or least idealized) level (see Table 1) because manipulations are directly perceivable to humans. The symbolic representations are placed on the other end of the continuum (i.e., least concrete or most idealized), and the micro level provides the intermediate bridge between the macro and the symbolic representations. Lin et al. (2016) examine the effects of four sequences of these three representations on students' ability to translate between chemistry representations when learning about phase changes. The concrete representation consists of an instructional video of water freezing and melting while the intermediate (i.e., micro-scale) representation is a video of an instructor interacting with a simulation representing the molecules' behavior in different states of matter. The phase changes are also represented symbolically by presenting the associated quantities (velocity, kinetic energy, and temperature). The results show, however, that the four sequences do not result in statistically significant differences in students' learning performance.

Even though Lin et al. (2016) align the different representations used in chemistry on a continuum of concreteness, they emphasize that the chemistry triplet comprises additional differences and should rather be perceived of as complementary because different aspects of the same phenomena stand out in each representation (cf. Ainsworth 2006). Moreover, the progression from top to bottom in the chemistry triplet is not simply about shedding irrelevant details; rather, the representations are on different levels of explanation. Learners need to understand each individual level, and they need to learn to translate between the representations for the different levels. As with the physics examples, the macro-level representations of chemistry are not arbitrary but specifically selected instantiations of a scientific model or theory. There is no obvious way in which the molecular model maps onto the macro level. Moreover, the micro-level representation is not simply a pictorial version of the macro level, as is the case in mathematics.

Students regularly have problems with specific external representations used in chemistry—such as interpreting equations as representing a static process rather than a dynamic one—as well as with translating between representations (Wu and Shah 2004). Understanding the representations and the translations requires conceptual knowledge about the underlying concepts and the representations themselves. In addition, several researchers suggest that visual and spatial reasoning may play an important role in chemistry learning and therefore propose that comparing visual features of multiple representations may support students' learning (Kozma 2003; Rau 2015; Wu and Shah 2004). The perceptual learning literature indicates that experiences leading to expertise can be divided into discovery and fluency. Discovery involves identifying the relevant category, whereas fluency entails the ability to use this information rapidly and more automatically with practice (Kellman and Garrigan 2009; Kellman et al. 2010). Experts have the skill of "just seeing" the relevant features of a representation (for an overview, see Chi and VanLehn 2012), including effortless noticing of connections between different representations. Rau (2015) suggests that training perceptual fluency through categorization tasks, referred to as the perceptual learning paradigm (Kellman et al. 2010), facilitates the learning of chemistry. Indeed, students benefit from rapid categorization tasks that require them to match a Lewis structure with the corresponding electrostatic potential map (Rau 2015). However, students also need sufficient understanding of the single representations to benefit from the training.

The chemistry triplet and the research on students' chemistry learning with multiple representations provide important qualifications for the evaluation of the generality of the concreteness fading approach. Even though there seems to be no straightforward mapping between the different chemical representations (e.g., Wu and Shah 2004), visual mapping of graphical representations and conceptual sense-making have been shown to benefit learning (Rau 2015; Kozma 2003). We suggest that two structure-mapping processes (Gentner 2010) might cause these benefits. First, conceptually linking the features between two representations may trigger re-representation, as when one moves from understanding of a chemical equation as a mere composition of letters and arrows to understanding it as an equation that represents the processes of a chemical reaction. Second, perceptual fluency training may support schema abstraction of shared features, making processing of the representations faster and helping students to generalize their knowledge to novel instances. The complementary roles of chemistry representations also imply that the relative effectiveness of concreteness fading is dependent on the nature of the representations. Seeing them as "one and the same," as described in theoretical accounts of concreteness fading (Fyfe and Nathan 2019, p. 417) cannot be the goal. Rather, students have to learn that the representations in chemistry are complementary and provide explanations on different levels. Thus, students need to learn the single representation and to conceptually link the representations as well as to understand these links.

Another feature of chemical representations is that they may illustrate emergent properties. That is, properties on the macro scale may depend on the interactions of the particles at the micro scale but differ from the properties of an individual particle (Talanquer 2011). A mapping between macro and micro levels spans different levels of explanation: some of the macro-level properties are contingent on the interactions at the lower level (Chi et al. 2012). Students often have misconceptions about how the macro-level features emerge. These aspects are prominent in understanding complex systems, which is important in chemistry, but even more prominent in biology education. Before turning to biology in the next section, let us summarize our analysis of chemistry. Apparently, external representations used in chemistry are conceptually different from representations used in mathematics. Representations in chemistry are not simply representations differing in concreteness; they are representations on different levels of explanation. They mutually inform each other, and learning chemistry requires an understanding not just of the single representations but also of their interdependencies.

Biology

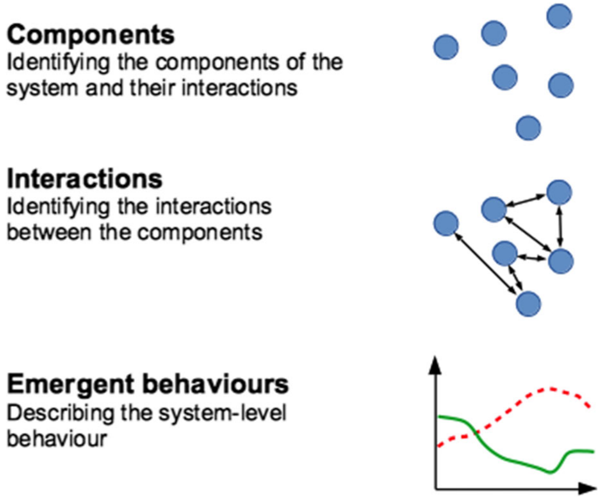
As with chemistry and physics, biology covers a wide range of levels of explanation, from the molecular and cellular levels to the level of the entire biosphere. Biological phenomena are often approached and investigated from the perspective of hierarchical or complex systems. This approach imposes high cognitive demands on learners because they have to make sense of the complex interdependencies and causalities not only within but also across levels (Chi et al. 2012; Goldstone and Wilensky 2008; Hmelo-Silver et al. 2007; Jacobson and Wilensky 2006).

A general framework for analyzing and interpreting these complex systems hierarchies is presented in Fig. 3. It conceptualizes three levels of complex systems: components, interactions, and emergent behaviors (Buckley and Quellmalz 2013). The different levels of such a multi-level system may align with the concrete, intermediate, and idealized levels in the concreteness fading sequence proposed by Lin and colleagues (Lin et al. 2016; see Table 1), with the biochemical level being the least concrete. However, rather than comprising a more idealized representation of macro-level phenomena, the biochemical level constitutes its own conceptual sphere, with its own phenomena. With regard to the levels in Fig. 3, the component level most readily corresponds to the concrete level because it captures the level at which phenomena are "experienced,"[1] similar to the macro level in the chemistry triplet. For instance, Buckley and Quellmalz (2013) use an animation with a life-like depiction of organisms (e.g., grass, lizards, crickets) to illustrate the different components of an ecosystem. The interaction level corresponds to a description or a dynamic model of the interactions between the components, often including significant amounts of idealization to capture relevant aspects of the phenomenon (e.g., an ecosystem). The emergent behavior level may consist of a mathematical description of, for instance, population dynamics devoid of contextual or component-level details.

The different levels in the systems described above seem to align well with the chemistry triplet. However, Tsui and Treagust (2013) argue that chemistry representations stand for a specific entity whereas biology spans nested hierarchies of multiple entities—cells are nested within tissues, which are nested within organs, and so forth. Thus, while the examples of the

[1] One should bear in mind, however, that in biology, the macro level is sometimes used to refer to entire ecosystems out of reach of direct observation or experience. Therefore, researchers have suggested the additional level of the mesocosm, which refers to the phenomena at the human scale perceivable by our sensory abilities (Niebert et al. 2013)

Fig. 3 Different levels of a systems model (Buckley and Quellmalz 2013)



chemistry triplet discussed previously offer more or less complementary representations of the same phenomena (or processes), the complex systems framework emphasizes the idea that there exist different but related processes on different levels that cannot be captured by any single-level explanation.

One challenge to understanding such hierarchical systems is rooted in the counterintuitive nature of many emergent phenomena and the complex, often non-linear interaction patterns. It is difficult to relate the phenomenological mathematical representation to the actual organisms or components of the systems and their interactions. The macro-level representations, such as graphs or mathematical equations standing for the patterns of population dynamics, may still be relatively easy to grasp, but the non-linear and stochastic nature of many complex phenomena at the interaction level is harder to grasp. For example, the component- and interaction-level descriptions of different ecosystems consider the organisms as constituents of an ecosystem, its relations, and its interactions. These cross-level relations are not present in the mathematical descriptions of the population-level dynamics, meaning that predicting or linking these levels is opaque. One cannot infer the complex nature of the system from the mathematical description of the population-level dynamics. Moreover, similar behaviors can arise from fundamentally different agent-level interactions. Many students think that the macro-level behavior of such a hierarchical system exhibits linear effects resulting from centralized control with deterministic causality (Chi et al. 2012; Jacobson and Wilensky 2006). Students must overcome this naïve conception to grasp that higher-level processes emerge from simple lower-level interactions without causal agency.

Concreteness fading has been used in research into the learning of biological concepts, but there are fewer studies than for the other domains. In one study, Goldstone and Son (2005) apply the systems modeling approach to teach students about the way in which agents or component-level interactions produce emergent phenomena within biological systems. They use a two-step concreteness fading sequence to teach the principles of complex adaptive systems using two different simulations that represent ants foraging for food. In the concrete simulation, ants and food sources appear as relatively realistic line drawings. That is, the ants have legs and heads, and pictures of apples are shown as food sources. In the idealized simulation, the ants and food sources are idealized into dots and blobs, respectively. Goldstone and Son (2005) report that concreteness fading produces better learning and transfer performance than concreteness introduction does, and that both of these sequences produce better transfer performance than learning with only the idealized or the concrete simulation. They conclude that, first, presenting learners with multiple instantiations of the same concepts generally supports schema abstraction and, second, that concreteness fading helps to ground concepts (supporting elaboration with prior knowledge) but also provides an idealized, more transferable representation. Moreover, when examining the concrete-only and idealized-only conditions, they find that the simulation appearance differentially affects learning and transfer performance. The idealized-only simulation results in better transfer performance while the concrete-only simulation results in better performance in the simulation itself. In other words, the concrete-only condition may result in a richer internal representation of the individual simulation (i.e., by activating relevant prior knowledge) whereas the idealized representation leads to the construction of a knowledge representation that can more easily be applied to novel problems (i.e., a more schematic internal representation).

Much like the DC circuit representations reported in the section on physics, the simulations used by Goldstone and Son (2005) provide a close match and perceptual linkages to facilitate the mapping between them. However, this experiment in the domain of biology provides positive evidence for the benefits of concreteness fading that was not found in the experiment with DC circuit representations (Jaakkola and Veermans 2018). It is likely that the perceptual support (i.e., concrete display of ants) helps students to make better use of their intuitive prior knowledge. The authors note that the anthropocentric interpretations (descriptions of the ants as "getting tired," "scaring other ants away," etc.) elicited by concrete simulations correlate with the true role-governed behavior of the ants (Goldstone and Son 2005, p. 99). By contrast, in the DC circuit experiment, the concrete and idealized simulations hardly vary along the dimension of concreteness. Rather, circuits are presented only in different contexts (bulbs vs. resistors), as resistors are not abstract depictions of bulbs in the same way that dots are abstractions of ants in the biology context; resistors are just different components to bulbs.

Agent-based simulations such as those used by Goldstone and Son (2005) have repeatedly been used to teach complex systems and the way in which changes on the individual level affect the population level (Cuzzolino, Grotzer, Tutwiler, & Torres, 22,019). There is some evidence that such agent-based modeling approaches support students' understanding of complexity, as students can use their prior knowledge and experience to make inferences about the behavior of agents and connect this to macro-level phenomena (Yoon et al. 2018). In the framework of structure-mapping theory (Gentner 2010), the agentive framing elicited by the concrete simulations supports inference projection to a more idealized simulation. However, even though agent-based approaches have been shown to support students' causal reasoning, the effect may be strongly dependent on the context in which the approaches are used (Cuzzolino et al. 2019). The concrete simulations with ants arguably evoke relevant and useful prior knowledge, but more research is needed to determine whether and, if so, which sequences of representations provide such benefits in other contexts. In the vocabulary of the DeFT framework, different simulations may constrain the interpretation of successive examples (including in unintended ways) and hence it will be necessary to determine which kinds of representations students acquire when interacting with agent-based models. For instance, Cuzzolino et al. (2019) suggest that abstraction via contrasting cases that align or do not align with agentive bias may support students in acquiring knowledge about instances of complex systems that are more diverse. Despite these exciting directions for future research, it should be clear that external representations used in biology (like those used in physics and chemistry) do not map well onto concreteness fading sequences as they have been investigated in mathematics and that learners need to learn and understand all individual representations and their (complex) interrelations.

Discussion

Concreteness fading has been advanced as a domain-general approach to sequencing external representations. In a review paper about concreteness fading, Fyfe et al. (2014, p. 9) say this in their abstract:

> A longstanding debate concerns the use of concrete versus abstract instructional materials, particularly in domains such as mathematics and science. Although decades of research have focused on the advantages and disadvantages of concrete and abstract materials considered independently, we argue for an approach that moves beyond this dichotomy and combines their advantages. Specifically, we recommend beginning with concrete materials and then explicitly and gradually fading to the more abstract.

In a similar vein, Fyfe and Nathan (2019, p. 404) propose "concreteness fading as a theory of instruction that has broad application." To be clear, we see value in concreteness fading in specific contexts and domains; that is, specifically, for learning basic mathematical concepts. Moreover, we have examined a specific operationalization of concreteness fading suggested by Fyfe and Nathan (2019) along with studies some of which do not rigorously align with this operationalization (but which were labeled as concreteness fading by their authors). Our analysis is qualified by the scope of the specific operationalization, which largely sees concrete representations as enactive, physical objects (or representations resembling them) from which to start instruction and fade concreteness. That is, in subsequent steps, contextual detail is to be eliminated and finally the principle to be learned is provided as a highly idealized, often symbolic representation (Fyfe and Nathan 2019). Alternative definitions, operationalizations, and characterizations are certainly possible. For example, on the basis of the biology example involving different kinds of simulation (Goldstone and Son 2005) or the physics examples with the different DC circuits (Jaakkola and Veermans 2018), one could derive a concreteness fading definition with closer mappings between the representations. One would not then necessarily fade from enactive representations (e.g., experiments) to purely symbolic representations (e.g., laws expressed in mathematical formula) but switch from a slightly more concrete to a slightly less concrete representation by using similar representations and thus try to provide mappings that are easier to recognize (given the overlapping perceptual features). Whether such subtle changes in concreteness are generally effective seems unlikely, however, given the empirical results we reported in our conceptual analysis of the different domains. Thus, we doubt that concreteness fading can be a promising domain-general instructional approach. Our doubt is general, in that it is not strictly dependent on a specific definition of concreteness fading. Rather, we question the simplistic idea of fading concreteness over a sequence of representations based on the single factor of concreteness alone.

Our doubts result from the presented conceptual analysis of existing and potential implementations of concreteness fading in different domains. To conduct this analysis, we began by specifying what external representations are, describing how they can be categorized, and noting that educational settings typically make use of MERs. Subsequently, we presented examples from mathematics, physics, chemistry, and biology education to scrutinize the implementation of the concreteness fading approach within these domains. We can now bring these analyses together to evaluate the generality of the concreteness fading approach across domains. We suggest that the generalizability of the concreteness fading approach, and probably of any related instructional approach, depends on (at least) two aspects. First, it depends on the similarity of the external representations used and their relations to each other, both within domains and across domains. Second, it depends on which learning processes should be triggered by the different representations to reach a specific instructional goal.

Similarity of External Representations and Their Relations to One Another

Concreteness fading could function as a general instructional approach if the representations used in various domains were similar and had similar relations to each other. Our conceptual analysis indicates that this precondition is not fulfilled. In mathematics, one is rather free to select and construct the concrete representations: numerals can be represented by any set of objects, for instance. The concrete representation only has the function of supporting the understanding of the target concept as expressed by the idealized representation, typically, some form of mathematical symbolism. Thus, the teacher can construct the concrete representation specifically to support the learning of the idealized target representation. The learner need not remember the particular concrete representation used to teach equivalence or numerals (e.g., a balance beam or apples). In physics, biology, and chemistry, concrete representations are not (as) arbitrary. The concrete representations are specific instantiations of a particular phenomenon, principle, or law. For example, typical school experiments in physics are, in some sense, abstractions, but they are still rich in contextual detail, and, at the same time, educators usually have relatively little freedom when constructing them (they are ingenious settings that are often regarded historically as scientific breakthrough experiments). Moreover, a concrete representation, such as a hands-on experiment in physics, a specific macroscopic chemical reaction, or a study of a certain species in biology, often becomes meaningful for students only after the acquisition of some knowledge of the idealized principle or law that the concrete representation instantiates. Moreover, different kinds of representations often mutually inform each other—they are complementary. A less concrete (or more idealized) representation, such as a force diagram in physics, often contains or introduces additional relevant conceptual information—it does not just shed superficial information as would be the case in mathematics.

To repeat, our conceptual analysis indicates that, with regard to the general underlying idea of concreteness fading, there is a straightforward analogical mapping in mathematics but not necessarily in physics, chemistry, and biology. The representations described in the chemistry triplet are not analogous and do not align well on a single continuum. Rather, they constitute different levels of explanation and provide complementary information. This statement applies to biology as well. Ecological systems are frequently modeled as complex systems constituting a hierarchy in which different representations capture different levels of explanation rather than constituting a progression of analogous external representations in which perceptual information is stripped away over subsequent steps of idealization. These differences reflect the different functions MERs can have, such as providing different pieces of information or supporting abstraction (cf. Ainsworth 2006). Thus, our analysis does not provide new insights into the use of MERs but highlights that these differences are important aspects to consider when attempting to generalize an instructional approach.

Learning Processes and Instructional Goals

Concreteness fading could be considered as a general instructional approach if educators would want to trigger the same learning processes when using MERs to reach similar instructional goals. Our conceptual analysis indicates that this precondition is not fulfilled.

One of the main ideas of concreteness fading is to help learners activate or acquire meaningful and relevant prior knowledge by using concrete representation(s) as an initial instructional step (Fyfe and Nathan 2019). This knowledge can then be used to deal with and interpret external representations that are more idealized in subsequent instructional steps. As described in the previous section, the concrete, intermediate, and idealized external representations are typically highly analogous in the domain of mathematics. This relational similarity sets the stage for structural mappings across representations. For example, representing mathematical equivalence with a balance scale may help to conceptualize equivalence as balance. The balance scale serves as a base analog for understanding the mathematical principle of equivalence and its mathematical notation, the target analog. The initial understanding achieved with the enactive balance scale representation may then be carried through the examples via inference projection. The shedding of details in the fading sequence may support students in acquiring a generalized schema of the target concept (Gentner 2010).

Some examples in other natural science domains discussed above provide obvious alignable similarities, too—for instance, different representations of DC circuits or chemical structures (see Jaakkola et al. 2010; Johnson et al. 2014; Rau 2015). This similarity may invite analogical comparison, which promotes the learning of an abstract schema and inference projection from one representation to another. Comparing instances of electric circuits or molecular representations may also support learning by eliciting re-representation. Simulations, for instance, may offer additional details supporting re-representation by enriching the learner's internal representations of the target concepts.

Re-representation and schema abstraction via visual mapping and comparison may be important for learning scientific concepts, but the relationships between MERs in these domains are typically more complex. The typical concrete representations in physics, chemistry, and biology are important in themselves and often provide complementary information. Hence, the learning goals also differ from learning goals in mathematics. In mathematics, the concrete and intermediate representations are a way to learn the more idealized principle, whereas in physics, chemistry, and biology, it is necessary to understand and learn the individual external representations and how to link them conceptually across levels.

The complementarity may invite learning via re-representation. In physics, recognizing a problem or a phenomenon as an instance of a particular principle or law may trigger re-representation of the problem, which may subsequently help in seeing the problem on the basis of the deep structure specified in a principle or law (Chi and VanLehn 2012). Likewise, linking partial pressure curves with an animation of a chemical process (Kozma 2003) may result in re-representation. Students may develop internal representations of the curves and animations as instances of target concepts, such as the concept of equilibrium. In biological ecosystem models, students may need to learn how to map the features of a component-level representation with a mathematical description of the system-level dynamics: The numbers of particular species may map onto a specific dimension of a graph, for instance, potentially causing re-representation of the ecosystem concept.

The complementarity of the different-level representations again indicates that Fyfe and Nathan specified "real goal" of concreteness fading, that is "to think of the representations as one and the same" (2019, p. 417), is not the goal in many domains and contexts. Rather, learners need to understand the complementarity and how different-level representations highlight different aspects of the target phenomenon or concept. The complementarity also brings into focus the fact that different MERs afford different reasoning processes. Different-level representations may not be perfectly alignable, but they may still constrain their mutual interpretation. For instance, the qualitative analysis of students' transfer performance in Goldstone and Son (2005) implies that the agent-based simulations of biological system invoked anthropocentric interpretations. While such interpretations may result in better transfer in some contexts, they may also elicit interpretive bias, which is harmful in other contexts.

On a more general note, our conceptual analysis highlights the fact that claims about the broad applicability of concreteness fading, or any instructional approach for that matter, requires domain-specific knowledge of the various target domains (biology, physics, chemistry, etc.), knowledge about differences between domains, and knowledge about subject-specific education. Many studies in educational psychology focus on mathematics. Arguably, there are good reasons for this focus. Mathematics allows for the relatively easy construction of tasks of varying difficulty by changing aspects of the context, as well as the construction of perfectly isomorphic examples, which is not necessarily possible in other domains. Given these domain-dependent aspects, we believe researchers have to be cautious about overgeneralizing instructional approaches. Before calling for empirical research to test an approach in different domains, it is necessary to conceptually analyze whether an approach fits the domain-specific characteristics and learning goals. Our conceptual analysis indicates that concreteness fading may not have the broad applicability that has been suggested by the literature.

Conclusion

We have identified profound differences between mathematics and the natural sciences with regard to the external representations used and the intended learning processes to be triggered by MERs. Finding these differences required expertise in mathematics, physics, chemistry, and biology as well as in educational and psychological research. It is rare to find someone with this combination of expertise. One of the authors of this conceptual analysis is a cognitive psychologist who investigates STEM education—let us call him A— and the other author is a physics education specialist—let us call him B. B once told A that students in physics always struggle to understand topic X and A suggested the use of concreteness fading. Together, they tried to design the appropriate materials but ran into trouble and wondered why. They talked to mathematics, chemistry, and biology education experts working in higher education. All initially liked the idea of concreteness fading but also ran into trouble when we asked them to design a lesson in which they would implement the idea of concreteness fading for relatively advanced concepts. We decided to dig deeper and conduct this conceptual analysis to identify the source of the problems involved in implementing the concreteness fading approach across domains. This analysis would not have been possible without the support of our colleagues—it is a truly interdisciplinary effort. Such an interdisciplinary effort is necessary if the aim is to develop and promote a domain-general instructional approach.

Authors' Contributions Both authors contributed equally to this article.

Funding Open access funding provided by University of Helsinki including Helsinki University Central Hospital. Tommi Kokkonen was funded by grants from the Academy of Finland (grant no: 311449) and the Eino Jutikkala Foundation.

Compliance with Ethical Standards

Conflict of Interest The authors declare that they have no conflict of interest.

Data Availability Not applicable.

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